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Question Paper Code : 20754

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering, Information Technology)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

(Use of standard normal distribution table may be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Check whether the function given by $f(x) = \frac{x+2}{25}$ for $x = 1, 2, 3, 4, 5$ can serve as the probability distribution of a discrete random variable.

2. A random variable X has the probability function $f(x) = \frac{1}{2^x}$ for $x = 1, 2, 3, \ldots$ Find its moment generating function.

3. Let X and Y have the joint probability mass function

		X		
		0	1	2
Y	0	0.1	0.4	0.1
	1	0.2	0.2	0

Find $P(X+Y>1)$ and $E(XY)$.

4. The joint probability distribution function of the random variable (X, Y) is given by $f(x, y) = k(x^3 y - xy^3)$, $0 \le x \le 2; 0 \le y \le 2$. Find the value of k.

5. Write the classification of random processes.

6. State any two properties of Poisson process.

7. State the basic characteristic of queueing system.

8. Write the Little's formula for queueing system.

9. State the formula for the probability that there are n customers in the system of (M/M/1) :(FIFO/N/∞).

10. Define: Open Jackson networks.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) A radar system has a probability of 0.1 of detecting a certain target during a single scan. Use Binomial distribution to find the probability that the target will be detected at least 2 times in four consecutive scans. Also compute the probability that the target will be detected at least once in twenty Scans. (8)

(ii) An electrical firm manufactures light bulbs that have a length of life which is normally distributed with mean $\mu = 800$ hours and standard deviation $\sigma = 40$ hours. Find the probability that a bulb burns between 778 and 834 hours. (8)

Or

(b) (i) The probability of an individual suffering a bad reaction from an injection of a certain antibiotic is 0.001. Out of 2000 individuals, use Poisson distribution to find the probability that exactly three suffer. Also, find the probability of more than two suffer from bad reaction. (8)

(ii) Electric trains in a particular route run every half an hour between 12. Midnight and 6 a.m. Using uniform distribution, find the probability that a passenger entering the station at any time between 1.00 a.m. and 1.30 a.m. will have to wait at least twenty minutes. (8)

12. (a) The joint probability distribution of a two dimensional random variable (X, Y) is given by $f(x,y) = \frac{1}{3}(x+y)$, $0 \le x \le 1$, $0 \le y \le 2$. Find the correlation coefficient. Also, find the equations of the two lines of regression. (16)

Or

(b) (i) Two random variables X and Y have the joint probability density function $f(x, y) = \begin{cases} 2 - x - y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$. Find the marginal densities of X and Y. Also, find the conditional density functions. (8)

(ii) The joint probability distribution of a two dimensional random variable (X, Y) is given by $f(x, y) = x + y$, $0 \le x \le 1$, $0 \le y \le 1$. Find the probability distribution function of $U = XY$. (8)

13. (a) (i) Show that the random process $X(t) = A\cos\omega t + B\sin\omega t$ is wide sense stationary process if A and B are random variables such that $E(A) = E(B) = 0$, $E(A^2) = E(B^2)$ and $E(AB) = 0$. (6)

(ii) A machine goes out of order whenever a component part fails. The failure of this part is in accordance with a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have lapsed since the last failure. If there are 5 spare parts of this component in an inventory and the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (10)

Or

(b) Consider a Markov chain on (0,1,2) having the transition matrix given by $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$. Show that the chain is irreducible. Find the period and the stationary distribution. (16)

14. (a) Derive the steady-state probabilities of the number of customers in M/M/1 queueing system from the birth and death processes and hence deduce that the average measures such as expected system size L_s, expected queue size L_q, expected waiting time in system W_s and expected waiting time in queue W_q. (16)

Or

(b) A petrol pump station has 4 pumps. The service time follows the exponential distribution with a mean of 6 min. and cars arrive for service in a Poisson process at the rate of 30 cars per hour. What is the probability that an arrival would have to wait in line? Find the average waiting time in the queue, average time spent in the system and the average number of cars in the system. (16)

15. (a) Derive the Pollaczek — Khinchin formula for M/G/1 queue. Hence deduce the result for the queues M/D/1 and $M/E_k/1$ as special cases. (16)

Or

(b) Consider a system two servers where customers arrive from outside the system in a Poisson fashion at server 1 at a rate of 4 / hour and at server 2 at a rate of 5/hour. The customers are served at station 1 and station 2 at the rate of 8 / hour and 10 / hour respectively. A customer, after completion of service at server 1 is equally likely to go to server 2 or to leave the system. A departing customer from server 2 will go to server 1 twenty five percent of the time and will depart from the system otherwise. Find the total arrival rates at server 1 and server 2. Find the limiting probability of n customers at server 1 and m customers at server 2. Find the expected number of customers in the system.
